ASYMMETRIC POLICY EFFECTS, CAMPAIGN CONTRIBUTIONS, AND THE SPATIAL THEORY OF ELECTIONS

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Abstract—A spatial model of elections with campaign contributions is constructed in which contributors give money to help the candidates they like get elected. It is shown how candidate-specific policy effects on firms cause candidates to adopt different policy positions. It is also shown how the additional presence of firm-specific policy effects may cause polarization of candidate policy positions. A comparative statics analysis establishes relationships among several key parameters of the model. Even though contributors take candidate positions as given, anticipatory position-taking by the candidates causes contributors to exert a powerful influence over candidate behavior.

INTRODUCTION

A basic conclusion from the spatial theory of elections is that when two candidates compete for office, the desire to win causes them to adopt similar, if not identical, policy positions. In the one dimensional deterministic Black-Downs model, this conclusion takes the form of the median voter result. In the multidimensional deterministic model, optimal candidate positions are frequently nonexistent. Still, candidates who eliminate dominated policies must adopt a position in the uncovered set [Cox, 1989; McKelvey, 1986], a subset of the Pareto set that shrinks to the core as the distribution of voter ideal points approaches a core configuration.

In probabilistic voting models, where equilibria are more plentiful, convergent candidate equilibrium is the norm [Hinich *et al.*, 1972; Coughlin and Nitzan, 1981; Enelow and Hinich, 1984]. If candidates are uncertain about voter preferences and maximize the probability of winning, they will adopt positions very close to each other [Glazer *et al.*, 1989]. Thus, candidate uncertainty about the vote causes two candidates to stick close together.

If campaign contributions are introduced into the spatial model, the linkage between the desire to win and candidate policy convergence poses an obvious question for a policy-based theory of campaign contributions. If both candidates in an election adopt the same policy, why should contributors give money to either candidate?

It is possible to argue that contributors give money for reasons other than policy, such as private benefits. Still, empirical work shows that campaign contributions by Political Action Committees are strongly related to policy proximity between contributor and recipient [Poole and Romer, 1984]. Summarizing the political science literature, Sorauf [1984, p. 339] suggests that PACs commonly "desire to elect public officials with values and preferences that promise a sympathy for the goals of the contributor."

If a contributor gives to a candidate only if he has a more appealing policy position than his opponent, what accounts for this policy difference? One possibility is that candidates adopt different policies in the absence of contributions. Another possibility is that the quest for campaign contributions causes candidates to cater to the policy wishes of contributors.

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Of course, there is more than one way contributions can affect policy. Most obviously, the contributor may bribe the candidate to adopt a policy position more to the contributor's liking, a practice that is illegal in American elections. The other possibility is that candidates go fishing for contributions, adopting positions designed to elicit financial support. This anticipatory position-taking is a decision on the part of the politician to court contributions rather than an effort by the contributor to bribe the politician. The contributor, in turn, responds to the positions that are taken by supporting the politician of his choice.

Whether candidate policy positions are influenced by potential contributions is a highly controversial subject. On the one hand, politicians such as Sen. Robert Dole R-Kansas have remarked that "when these political action committees give money, they expect something in return other than good government" [Smith, 1988, p. 257]. On the other hand, Davidson and Oleszek [1985, p. 357] quote Rep. Stewart McKinney R-Conn. as stating about PACs: "They all give money, and everybody ignores them." However, this remark does not imply that legislators are indifferent to receiving money, only that contributions and policies are in some kind of equilibrium.

In this paper, we adopt the assumption that PACs try to help the candidates they like get elected, rather than engage in bribery. Then, if policy divergence is required for contributions to be made, we must explain how the expectation of money causes candidates to move apart if they would otherwise stick together. Put differently, if spatial theory predicts that two candidates will adopt the same policy position in the absence of contributions, how does allowing contributions pull candidates apart? This is the question this paper is designed to answer.

The literature on spatial models of elections with campaign contributions is both small and of recent vintage. A much larger literature exists on election models with contributions (e.g., [Barro, 1973; Ben-Zion and Eytan, 1974; Welch, 1974; Bental and Ben-Zion, 1975; Hinich, 1977; Aranson and Hinich, 1979; Chappell, 1981; Becker, 1983; Denzau and Munger, 1986; Snyder, 1989]), but these models are not explicitly spatial. That is, candidate policy positions, conceived as points in a Euclidean issue space, are not endogenously derived from the assumptions of the model. The much smaller literature which meets this criterion includes [Brock and Magee, 1978, 1980; Austen-Smith, 1987; Congleton 1987; and Edelman, 1988]. The models of Hinich and Munger [1988] and Ingberman [1989] are hybrid models, both spatial in design, but non-spatial in the sense that spatial location is not a strategy variable.

In the Brock and Magee [1978, 1980], Austen-Smith [1987], and Congleton [1987] models, candidates are trying either to maximize expected votes or probability of winning, while in the Edelman [1988] model, candidates maximize expected policy utility as in [Wittman, 1983]. Among the spatial models with contributions and win-seeking candidates, Brock and Magee [1978, 1980] do not have an individual-level model of voting decisions, while Congleton [1987] fails to arrive at any general results about where candidates will locate if contributions affect their positions.

This leaves [Austen-Smith, 1987] as the one article that models individual-level voter decisions, assumes win-seeking candidates, and derives equilibrium results about the spatial locations of the candidates when contributions affect these positions.

The Austen-Smith model has several other features that are worth noting. Each candidate is perceived as a random variable, voters are risk-averse, and vote probabilistically. Furthermore, the effect of campaign expenditures (which equal contributions) is to reduce the variance of the candidate variable. Thus, expenditures by a candidate have a strictly positive effect on the probability that any voter votes for the candidate (with this effect assumed to be marginally decreasing in expenditures). Contributions are made by firms, which take candidate positions as given, and make equilibrium contributions to the candidates in an effort to help the favored candidate win the election.

The model we construct in this paper departs from Austen-Smith's model in several respects. First, we assume probabilistic voting, but introduce the random element at the group rather than the individual level. Second, we do not take the "reduction of variance" approach assumed not only by Austen-Smith, but also by Hinich and Munger [1988] and Ingberman [1989]. This approach implies that, *ceteris paribus*, a candidate is always better off publicizing his policy position, no matter how distasteful it is to the voters.

Our approach is to assume that each candidate uses contributions to convey favorable non-policy information to the voters. The voters may already possess a certain stock of nonpolicy information about the candidates, both positive and negative. In addition, voters also care about policy issues, although we make no assumption about the relative weight voters place on policy versus nonpolicy issues. The purpose of spending money is simply to convince the voter that there are enough good qualities about the candidate to support him.

In our model, candidates converge in the absence of contributions. This means that contributions are the sole cause of candidate divergence. How do contributions cause divergence? As we will see, the answer depends on the relationship between incremental changes in candidate policy and anticipated marginal changes in contributions. Specifically, we show how asymmetric candidate effects on the expected profits of contributors cause candidate policy divergence. We also show how asymmetric firm effects cause candidate policy positions to move in opposite directions.

In the following section, we lay out the elements of our model. We then prove two equilibrium theorems about the candidates and the firms. We also provide necessary conditions for candidate divergence and positive contributions. The relationship between policy effects on firms and contribution effects on candidate spatial locations is also established. Several examples are provided to illustrate the equilibrium solutions of the model. We also provide a comparative statics analysis of several of the model's key parameters. A final section states our conclusions.

THE MODEL

In our model there are two candidates 1 and 2 and two firms A and B. The term "firm" should be intepreted generically to mean any large contributor interested in net political benefits, such as a PAC acting on behalf of its members. While federal laws limit PAC contributions, the absence of a budget constraint in our model can be justified either by interpreting the firm as a collection of like-minded PACs or interpreting contributions as including goods and services, independent spending, honoraria, or other devices such as party committees that allow full or partial avoidance of federal restrictions.

We assume a two-stage game of the type described by Brock and Magee [1978, 1980], Austen-Smith [1987], and Edelman [1988], with the candidates acting as Stackelberg leaders vis-a-vis the firms. The candidates move first, playing a Cournot-Nash game against each other, with each candidate anticipating the contributions that the firms will make contingent on the policy position each candidate adopts. The firms then move second, taking the candidate positions as given and choosing the optimal contribution levels in a second Cournot-Nash game of firm A against firm B.

Since we assume expected plurality-maximizing candidates, it makes no sense to assume that the firms move first, since the firms will recognize that candidates will simply take firm contributions and do what they otherwise would have done. Consequently, the firms will contribute nothing. On the other hand, PAC contribution data on U.S. House races provide circumstantial support for the proposition that firms do act as second movers [Edelman, 1988].

In keeping with the earlier models cited above, we assume that candidate policy positions are located on a compact, convex subset of the real line. For convenience, we assume each policy position can be represented as a nonnegative real number. Let t^1 denote candidate 1's position and t^2 candidate 2's position. Each candidate selects his position in an attempt to maximize his expected plurality. Each firm maximizes expected profits by trying to increase the probability that its favorite candidate wins the election. This is done through campaign contributions.

The voting model we employ is an expansion of the one used by Enelow and Hinich [1984]. There are k voting blocs, $i = 1, \ldots, k$, where the voters in each bloc agree about the measurable factors relevant to a voting decision. Let $u^i(t^1)$ and $u^i(t^2)$ represent the policy utility of each member of bloc i for candidates 1 and 2. Let $k^{i1}(c^1)$ and $k^{i2}(c^2)$ represent the nonpolicy utility of a member of bloc i for the two candidates, where c^1 and c^2 are the contributions made by firms A and B to candidates 1 and 2. Our decision to make each candidate's nonpolicy utility vary only with changes in his own contributions is a simplifying device. The results we obtain are sufficiently different from previous studies to justify this limitation. We do not assume that $k^{i1}(0)$ or $k^{i2}(0)$

need equal zero, meaning that the voters can possess nonpolicy information about the candidates before a dollar is spent. Nonpolicy information refers to such characteristics as the candidate's personality, experience, career background, and other fixed attributes that are orthogonal to his policy positions. These factors can be "negatives" as well as positives, such as a candidate's prior history of drug use or association with a scandal of some type.

The voting model we employ postulates that a member of bloc i votes for candidate 1 if and only if

$$d^i = \Delta k^i + \Delta u^i > \epsilon^i, \tag{1}$$

where $\Delta k^i = k^{i1}(c^1) - k^{i2}(c^2)$ and $\Delta u^i = u^i(t^1) - u^i(t^2)$ and votes for candidate 2 otherwise. The right-hand side of (1) is assumed to be an unobservable random variable across the members of bloc i, so the proportion of bloc i that votes for candidate 1 is $F^i(d^i)$, where F^i is the distribution function of ϵ^i. We can view ϵ^i as a random threshold level, whose distribution is known to the candidates and firms and which is uncorrelated with d^i. If ϵ^i is positive, the voter has a positive bias towards candidate 2; if ϵ^i is negative, the bias is towards candidate 1; and if ϵ^i is zero, no candidate has a built-in advantage.

Everyone votes (or a fixed, known percentage of the electorate votes), so maximizing expected plurality is equivalent to maximizing expected vote [Aranson, Hinich, and Ordeshook, 1974]. Candidate 1's objective function can then be written as

$$EV^1 = \sum_{i=1}^{k} N^i F^i(d^i). \tag{2}$$

Candidate 2's objective function is $EV^2 = \sum_{i=1}^{k} N^i[1 - F^i(d^i)]$. N^i is the positive fraction of the electorate contained in bloc i so $\sum_{i=1}^{k} N_i = 1$.

We assume that for each voting bloc i, F^i, u^i, and k^i are continuous and twice differentiable. We also assume that both u^i and k^i are strictly concave functions and that k^{ij} is a nonnegative, increasing function of c^j ($j = 1, 2$). Thus, contributions to a candidate are assumed to have a positive but marginally decreasing effect on the voter's valuation of him. In addition, we assume that the density of the distribution function is positive for all feasible d^i.

We now turn to a closer examination of the assumptions about firms A and B. Both firms wish to maximize expected profits, which depend on the policy positions taken by the candidates and the contributions made by the firms. As in [Brock and Magee, 1978, 1980] and [Austen-Smith, 1987], let firms A and B have diametrically opposed interests. As in these studies, assume that firms do not vote, and voters do not make contributions. Since we are assuming that t^1, $t^2 \geq 0$, we might interpret t as a tax rate as in [Austen-Smith, 1987], a tariff rate as in [Brock and Magee, 1978, 1980], or a spending issue. Firm A's ideal point is zero, and firm B's is either the maximum value of t or positive infinity. The policy space may be a more general dimension, such as political ideology. In this case, recalling that the policy space is compact, we can view zero as a lower bound on the set of feasible candidate positions.

A given policy may have a range of effects across voters and firms. First, the same policy may affect firms differently from voters. Second, the same policy may affect one firm differently from the other, and, third, the effect on either firm of implementing a given policy may depend on the candidate who implements it. The analysis of this third effect is the major innovation of this paper. For example, if t is a tax policy, the existence of fixed differences between the candidates on other dimensions (e.g., party affiliation or political experience) may mean that the effect on either firm of implementing t may depend on the candidate who implements it. We refer to this type of policy effect as *candidate-specific*. Policy effects which differ between firms are labeled firm-specific.

That the same policy can affect firms differently is a common finding in the public policy literature [Stigler, 1971; Wilson, 1980; Leone, 1986]. However, it is also known that the same policy may be enforced differently from one Administration to the next [Moe, 1982]. Deregulation and environmental policy are two areas where Administration-specific effects have been found [Ackerman and Hassler, 1981; Noll and Owen, 1983]. Civil rights policy is another example.

To represent the differential policy effects outlined above, we assume that the firms employ a linear mapping function with a zero intercept term, similar to that of Enelow and Hinich [1984]. Unlike Enelow and Hinich, however, this map may be candidate-specific. Thus, the effect of candidate k's policy on firm A is $B^{Ak}t^k$, while the effect on firm B is $B^{Bk}t^k$ ($k = 1, 2$; B^{Ak}, $B^{Bk} > 0$). B^{jk} is the marginal policy effect of t on firm j ($j = A, B$) given the election of candidate k. There are four possible policy weights (B^{A1}, B^{A2}, B^{B1}, B^{B2}), based on firm-specific and candidate-specific policy effects on the firms. The assumption of constant marginal policy effects can be justified by viewing the firms as taking a least-squares approach to estimating policy effects. Ingberman [1989] assumes different absolute, marginal benefits to contributors as the winning policy position changes, which is equivalent to postulating firm-specific policy effects. Austen-Smith [1987] also assumes firm-specific policy effects.

If t^k is candidate k's political ideology, we can tell a different story. Given the obvious disincentives for most voters to acquire information about candidates, it is reasonable to assume that the typical voter has only a general picture of candidate k, as represented by t^k. The firm, on the other hand, has a more specialized interest in the candidate. Rather than being concerned with an overall measure, such as k's ideology, firm j may translate k's ideology into a predicted position on an issue of special importance to the firm. This predicted position would be $B^{jk}t^k$.

In any event, voters are just as rational as firms. They simply face a different decision environment with different incentives. Recall that voters do process candidate-specific information through the additive nonpolicy utility terms k^{i1} and k^{i2}. As shown in [Enelow, Hinich, and Mendell, 1986], this additive policy-nonpolicy approach predicts individual vote choices just as well as an approach that also allows for interactive effects.

We now state firm A's objective function as the maximization of expected profits, or more generally, net expected benefits:

$$\pi^A = r(-B^{A1}t^1) + (1 - r)(-B^{A2}t^2) - c^{A1} - c^{A2}, \tag{3}$$

where r is the probability that candidate 1 wins the election, c^{A1} is the contribution made by A to candidate 1 and c^{A2} is the contribution made by A to candidate 2. Firm B maximizes the expected profit or net expected benefit function

$$\pi^B = rB^{B1}t^1 + (1 - r)B^{B2}t^2 - c^{B1} - c^{B2}, \tag{4}$$

where c^{B1} and c^{B2} are the contributions made by B to candidates 1 and 2. The space of feasible contributions is assumed to be compact and convex for each firm. As the number of voters approaches infinity, maximizing the probability of winning the election is equivalent to maximizing the expected vote proportion [Hinich, 1977], so as in [Austen-Smith, 1987], we can substitute EV^1 for r.

The following lemma follows from a straightforward application of the Kuhn-Tucker necessary conditions for an expected profit-maximizing solution to (3) and (4) (see [Brock and Magee, 1980; and Austen-Smith, 1987], so it will be stated without proof.

LEMMA 1. If $B^{A2}t^2 \geq B^{A1}t^1$, $c^{A2} = 0$; while if $B^{B2}t^2 \geq B^{B1}t^1$, $c^{B1} = 0$. If $B^{A2}t^2 = B^{A1}t^1$, $c^{A1} = 0$; while if $B^{B2}t^2 = B^{B1}t^1$, $c^{B2} = 0$.

The lemma states that if a firm contributes at all, it contributes to the candidate whose effective policy is closer to the firm's ideal point. If the two candidates' effective policies are the same, the firm contributes nothing at all.

To ensure that the derivatives of k^{i1} and k^{i2} are positive, we henceforth require that $B^{A2}t^2 \geq B^{A1}t^1$ and $B^{B2}t^2 \geq B^{B1}t^1$. This assumption implies some restrictions on candidate mobility and relative policy effects. It means that neither candidate can leapfrog the other, the same assumption made by Downs, [1957]. Shepsle and Cohen, [1990] describe "no leapfrogging" as a halfway house between total spatial mobility and complete immobility and view it as a reasonable assumption about real elections. Given this assumption, A will give no money to candidate 2 and B will give no money to candidate 1. To simplify notation, we now set $c^{A1} = c^1$ and $c^{B2} = c^2$.

EQUILIBRIUM CONDITIONS

Equilibrium in the firm game is shown by proving the concavity of the firm objective functions with the candidate strategies taken as fixed. Continuity of the firm objective functions follows from the continuity of F^i and k^i. Compactness and convexity of the strategy space is assumed. To ensure equilibrium in the candidate game, given equilibrium in the firm game, it is sufficient to show that each candidate's objective function is strictly concave in his own strategy variable, with the firms' best response functions substituted into the candidate objective functions. Each candidate objective function is continuous since it is the sum of continuous functions. Compactness and convexity of the strategy space is assumed.

Uniqueness of the equilibrium follows if the Jacobian of the game is negative quasi-definite [Friedman, 1986, pp. 44–46]. This condition is met if the second cross partial derivatives of the two candidates or firm objective functions are the negative of each other.

Throughout this section and the remainder of the paper, we will use subscripts to denote partial derivatives. For example, EV_{t1}^1 is the partial derivative of EV^1 with respect to t^1 and EV_{t1t2}^1 is the second cross partial with respect to t^1 and t^2.

Sufficient conditions for the existence of equilibrium in the firm game are stated in the following theorem. The proof appears in the Appendix. In the case of a linear distribution function, existence follows immediately from the strict concavity of the k^{ij} functions.

THEOREM 1. *Suppose*

$$-\left[\frac{k_{c1c1}^{i1}}{(k_{c1}^{i1})^2}\right] > \left[\frac{f_{di}^i}{f^i}\right], \qquad \text{for each } i = 1, \ldots, k \tag{5}$$

and

$$-\left[\frac{k_{c2c2}^{i2}}{(k_{c2}^{i2})^2}\right] > -\left[\frac{f_{di}^i}{f^i}\right], \qquad \text{for each } i = 1, \ldots, k. \tag{6}$$

Then firms A and B possess equilibrium strategies c^{1*} and c^{2*}. Furthermore, if F^i is linear, or $(B^{A2}t^2 - B^{A1}t^1) = (B^{B2}t^2 - B^{B1}t^1)$, these strategies are unique.

The following theorem establishes sufficient conditions for equilibrium in the candidate game. The proof also appears in the appendix. In conditions (7) and (8), f^i is the density of the distribution function F^i and f_{di}^i is the slope of the density at d^i. Since there cannot be equilibrium in the candidate game without equilibrium in the firm game, it is assumed in Theorem 2 that (5) and (6) hold.

THEOREM 2. *Suppose*

$$-\left[\frac{d_{t1t1}^i}{(d_{t1}^i)^2}\right] > \left[\frac{f_{di}^i}{f^i}\right], \qquad \text{for each } i = 1, \ldots, k \tag{7}$$

and

$$\left[\frac{d_{t2t2}^i}{(d_{t2}^i)^2}\right] > -\left[\frac{f_{di}^i}{f^i}\right], \qquad \text{for each } i = 1, \ldots, k. \tag{8}$$

Then candidates 1 and 2 possess unique equilibrium strategies t^{1*} and t^{2*}.

Since the candidates anticipate equilibrium firm contributions, $d_{t1}^i = k_{c1}^{i1} c_{t1}^1 - k_{c2}^{i2} c_{t1}^2 + u_{t1}^i$, where the partial derivatives of c^1 and c^2 with respect to t^1 incorporate the firms' best response functions. If the candidates were unaware of the effect of their positions on contributions, these partial derivatives would be zero. Differentiating again with respect to t^1,

$$d_{t1t1}^i = k_{c1c1}^{i1} (c_{t1}^1)^2 + k_{c1}^{i1} c_{t1t1}^1 - k_{c2c2}^{i2} (c_{t1}^2)^2 - k_{c2}^{i2} c_{t1t1}^2 + u_{t1t1}^i.$$

The first and second direct partials of d^i with respect to t^2 are similar.

If f^i is the normal density with zero mean, the right-hand side of (7) equals $-d^i/\sigma^{i2}$, where σ^{i2} is the variance of F^i. Thus, as σ^{i2} increases, conditions (7) and (8) become easier to satisfy. If F^i is linear, the right-hand side of (7) and (8) equals zero. Then, while the assumption that u^i and k^i are strictly concave does not guarantee that the left-hand side of (7) and (8) is positive, this assumption does make (7) and (8) more likely to hold.

FIRST ORDER CONDITIONS AND CANDIDATE CONVERGENCE

Given the satisfaction of the second-order conditions (5)–(8), we can examine the candidates' first-order conditions to determine the properties of the candidates' equilibrium strategies. Differentiating EV^1 with respect to t^1 and setting this partial derivative equal to zero, we obtain

$$EV_{t1}^1 = \sum_{i=1}^k N^i f^i(d^i)[k_{c1}^{i1} c_{t1}^1 - k_{c2}^{i2} c_{t1}^2 + u_{t1}^i] = 0. \tag{9}$$

Likewise, differentiating EV^2 with respect to t^2 and setting this partial derivative equal to zero, we obtain

$$EV_{t2}^2 = \sum_{i=1}^k N^i f^i(d^i)[k_{c2}^{i2} c_{t2}^2 - k_{c1}^{i1} c_{t2}^1 + u_{t2}^i] = 0. \tag{10}$$

Turning to the firms, their first-order conditions for c^{1*} and c^{2*} are given by the equations

$$\pi_{c1}^A = \sum_{i=1}^k N^i f^i(d^i)(B^{A2}t^2 - B^{A1}t^1)k_{c1}^{i1} = 1, \tag{11}$$

$$\pi_{c2}^B = \sum_{i=1}^k N^i f^i(d^i)(B^{B2}t^2 - B^{B1}t^1)k_{c2}^{i2} = 1. \tag{12}$$

A firm's equilibrium contribution function can be derived from (11) or (12), given a specific functional form for k^{i1} or k^{i2}.

Since the candidates anticipate the equilibrium contribution levels by the firms, we can solve for c_{t1}^1, c_{t1}^2, c_{t2}^2, and c_{t1}^2 by totally differentiating the firms' first-order conditions with respect to t^1 and t^2.

The following theorem establishes sufficient conditions for candidate convergence. The proof is contained in the appendix.

THEOREM 3. *If* $B^{A1} = B^{A2}$ *and* $B^{B1} = B^{B2}$, $t^{1*} = t^{2*}$.

The consequence of convergent candidate equilibrium should be obvious from Lemma 1: the firms will contribute nothing.

In Enelow and Hinich's [1984] model, the linear maps of candidate issue positions are not candidate-specific. Theorem 3 therefore implies that candidate divergence (and firm contributions) can occur only if Enelow and Hinich's model is violated. In fact, if only voters made contributions, candidate-specific maps would also be a necessary condition for contributions to occur.

Firm-specific policy effects are neither necessary nor sufficient for firm involvement in the election. A candidate-specific policy effect on at least one firm is necessary for campaign contributions to occur. That is, the effect of a given policy on at least one firm must depend on the candidate who implements the policy.

COMPARATIVE STATICS RESULTS WITH EXAMPLES

To gain a better appreciation of the model, we solve for the candidates' equilibrium strategies and the firms' equilibrium contribution levels for one particular specification of the k^{ij} functions. To keep matters simple, assume a single voting bloc in the electorate and a uniform density f^i on some unit interval. Then, $N^i = f^i(d^i) = 1$, and the i superscript can be dropped.

In the examples, we set $k^j(c^j) = b^j \sqrt{c^j}$ $(j = 1, 2)$. Contributions influence the nonpolicy value of candidates 1 and 2 in substantially the same way, though for equal c^j the marginal effects of contributions differ if $b^1 \neq b^2$.

As an initial example, suppose that $b^1 = b^2 = 1$, $B^{A1} = .5$ and $B^{A2} = 1$, and $B^{B1} = 1$ and $B^{B2} = 1.5$. For the same c^j, the marginal effect of contributions is the same for both candidates. However, both firm-specific and candidate-specific policy effects exist. Candidate 2 is associated

with a larger marginal policy effect on both firms (for example, politicians of candidate 2's party may enforce policy more vigorously than politicians of candidate 1's party). Regardless of the winning candidate, the marginal policy effect is larger for firm B than it is for firm A.

To simplify notation, let $\hat{t}^1 = B^{A1}t^1$ and $\hat{t}^2 = B^{A2}t^2$, $\tilde{t}^1 = B^{B1}t^1$ and $\tilde{t}^2 = B^{B2}t^2$, $\Delta\hat{t} = B^{A2}t^2 - B^{A1}t^1$ and $\Delta\tilde{t} = B^{B2}t^2 - B^{B1}t^1$. Since $k^j = b^j\sqrt{c^j}$, $k_{c1}^1 = b^1/(2\sqrt{c^1})$ and $k_{c2}^2 = b^2/(2\sqrt{c^2})$. Substituting into A's and B's first order conditions (11) and (12) we solve for c^{1*} and c^{2*} to yield

$$c^{1*} = \frac{(\Delta\hat{t})^2 (b^1)^2}{4}, \qquad c^{2*} = \frac{(\Delta\tilde{t})^2 (b^2)^2}{4}.$$

Given the parameter values above, contributions for any specific values of t^1 and t^2 are easily calculated. For example, if $t^1 = .4$ and $t^2 = .6$ then $c^{1*} = .04$ and $c^{2*} = .06$. Since each firm's contribution depends on the position of *both* candidates, a change in either candidate's position affects both contributions. For instance, if t^1 increases to .5 while t^2 stays at .6, then c^{1*} falls to .03 while c^{2*} falls to .04. This example illustrates the following proposition, which holds with any number of groups (a proof is given in the Appendix):

PROPOSITION 1. *If F^i is linear, $\frac{\partial c^{1*}}{\partial t^1} < 0$, $\frac{\partial c^{1*}}{\partial t^2} > 0$, $\frac{\partial c^{2*}}{\partial t^1} < 0$, and $\frac{\partial c^{2*}}{\partial t^2} > 0$.*

This proposition indicates that a firm increases its contribution to its preferred candidate as that candidate approaches the firm's ideal position, and decreases its contribution as the candidate moves away. In addition, the firm increases the contribution to its more preferred candidate as the other candidate moves away (since the more preferred candidate becomes relatively more attractive) and decreases its contribution as the less preferred candidate moves closer.

We next determine the positions the candidates will take, given the parameter values above. Substituting the firms' contribution functions into the candidates' objective functions and proceeding as we did in the earlier sections yields the following first order conditions for the two candidates:

$$\frac{B^{B1}(b^2)^2}{2} - \frac{B^{A1}(b^1)^2}{2} + u_{t1} = 0, \tag{13}$$

$$\frac{B^{B2}(b^2)^2}{2} - \frac{B^{A2}(b^1)^2}{2} + u_{t2} = 0. \tag{14}$$

Suppose voters' utility is quadratic with an ideal point at .5, i.e., $u(t) = -(t - .5)^2$. Then $u_{t1} = -2(t^1 - .5)$ and $u_{t2} = -2(t^2 - .5)$, and substituting into (13) and (14)

$$t^{1*} = \frac{1}{4}\left[B^{B1}(b^2)^2 - B^{A1}(b^1)^2\right] + .5, \tag{15}$$

$$t^{2*} = \frac{1}{4}\left[B^{B2}(b^2)^2 - B^{A2}(b^1)^2\right] + .5. \tag{16}$$

These equations may be used to solve for equilibrium t^1 and t^2. Given the parameter values above, $t^{1*} = t^{2*} = 5/8$. Although the announced positions of the candidates are the same, the firms still give money because the policy effects of the two positions are different ($\hat{t}^1 = 5/16 \neq \hat{t}^2 = 5/8$ and $\tilde{t}^1 = 5/8 \neq \tilde{t}^2 = 15/16$). The equilibrium contributions are $c^{1*} = c^{2*} = 25/1024$.

In this example, in equilibrium, it happens that $k^1 = k^2$ and $u^1 = u^2$, so $d = \Delta k + \Delta u = 0$. Suppose candidate 1 contemplates moving to the left of his equilibrium position to the electorate's ideal point of .5. Compared with his equilibrium position, candidate 1 will raise his contribution level (Proposition 1) and increase his policy value to the voters. At $t^1 = .5$ and $t^2 = 5/8$, c^{1*} increases to .04 from $25/1024 = .02$ and $u(t^1)$ increases to 0 from $-1/64$. But this gain is more than offset because firm B reacts to candidate 1's action by increasing c^{2*} to .05. The net result is that d falls to $-.01$, so candidate 1 does worse than if he had stayed at his equilibrium position.

The previous example illustrates how firms contribute in the model even if candidates adopt the same policy positions. However, positions need not be the same nor need the firms contribute the same amount. For example, suppose $b^1 = 1$ and $b^2 = 2$, $B^{A1} = B^{B1} = .5$, and $B^{A2} = B^{B2} = 1.5$. In this case, there are candidate-specific policy effects but no firm-specific effects, with a larger

marginal policy effect associated with candidate 2. Using the same quadratic utility function, we solve to find $t^{1*} = 7/8$ and $t^{2*} = 13/8$. The candidates diverge not only in terms of policy effects on the firms but also in their announced positions. Firm A contributes the amount 1 to candidate 1 and Firm B contributes 4 to candidate 2.

A simple modification of the model offers one way to explore the differences between elections in which voting decisions are mainly determined by policy and those in which nonpolicy concerns predominate. Introduce *sensitivity weights* into the d function,

$$d^i = w^i \Delta k^i + v^i \Delta u^i,$$

with w^i and v^i nonnegative for all i. The sensitivity weights reflect the relative importance of policy and nonpolicy issues in the voting decisions of group i's members.

How does an increase in policy sensitivity affect contributions, positions, and expected votes? The presence of the sensitivity weights alters the formulae for c^{j*} and t^{j*}:

$$c^{1*} = \frac{w^2 (\Delta \hat{t})^2 (b^1)^2}{4},$$

$$c^{2*} = \frac{w^2 (\Delta \hat{t})^2 (b^2)^2}{4},$$

$$t^{1*} = \frac{1}{4} \frac{w^2}{v} \left[B^{B1}(b^2)^2 - B^{A1}(b^1)^2 \right] + .5,$$

$$t^{2*} = \frac{1}{4} \frac{w^2}{v} \left[B^{B2}(b^2)^2 - B^{A2}(b^1)^2 \right] + .5.$$

Contributions are scaled by w^2 while positions alter depending on the ratio w^2/v.

Assume the initial parameter values (i.e., $b^1 = b^2 = 1$, $B^{A1} = .5$ and $B^{A2} = 1$, and $B^{B1} = 1$ and $B^{B2} = 1.5$) with $w = 1$ but $v = .1$. The sensitivity weight on policy falls to 1/10 its initial value, and it would appear that contributions are unaffected since w is unchanged. However, candidate positions shift from 5/8 to 7/4, since w^2/v increases from 1 to 10, and the candidates move away from the voters' ideal point of .5. As a result, both contribution levels fall from 25/1024 to 49/256. If, instead, v increases to 10 (so $w^2/v = 1/10$) both candidates locate at 41/80, very close to the voters' ideal point of .5.

These examples illustrate two general results, which hold for any number of groups. Proofs of the following two propositions are given in the Appendix.

PROPOSITION 2. *If F^i is linear, contribution functions are independent of v^i.*

Let x vary continuously across elections, with $v^i = v^i(x)$ and $v_x^i > 0$ (i.e., an increase in x results in an increase in all v). The variable x might represent increased media focus on the policy side of the campaign or any exogenous factor that affects the level of policy concern among the voters. The existence of such a "priming" effect is demonstrated in [Iyengar and Kinder, 1987].

PROPOSITION 3. *If F^i is linear, utility is quadratic, and $t^1 < t^2$, then, if x increases, at least one candidate moves toward $\bar{X} = \sum N^i f^i v_x^i X^i / \sum N^i f^i v_x^i$, where X^i is group i's ideal point.*

Proposition 3 indicates at least one candidate moves toward the "center" of the electorate if v increases, where the center reflects relative group size, and the response of each group's policy weight to a change in x.

In the examples above with $v = .1$ and 10, d is zero because Δk and Δu are zero. Hence, the change in policy sensitivity changes neither candidate's expected vote. However, consider the example of divergent equilibrium offered earlier (i.e., with $t^{1*} = 7/8$, $t^{2*} = 13/8$). In this example $\Delta k = -3$ and $\Delta u = 9/8$, so $d = -15/8$. Suppose v (initially set to 1) increases to 10. Equilibrium positions change to $t^{1*} = 43/80$, $t^{2*} = 49/80$, and d increases to $-.9$. Hence, candidate 1's expected vote increases.

This example illustrates the next proposition, which holds for any number of groups (a proof is given in the Appendix). Again, let x vary continuously across elections, with $v^i = v^i(x)$ and $v_x^i > 0$ so that an increase in x results in an increase in v for all groups.

PROPOSITION 4. *Assume F^i is linear. Then if $t^1 = t^2$, $\frac{\partial EV^1}{\partial x} = 0$; if $\Delta u^i > 0$ for all i, $\frac{\partial EV^1}{\partial x} > 0$; if $\Delta u^i < 0$ for all i, $\frac{\partial EV^1}{\partial x} < 0$.*

This proposition indicates that if one candidate is closer to every group's ideal point, that candidate's expected vote increases if x increases. Otherwise, if utility is quadratic, a candidate's change in expected vote from an increase in x is a weighted average of changes in each group, with the weights determined by relative group size and group sensitivity to changes in x.

Given that the candidates are playing a concave/convex game, divergent equilibrium requires that $EV^1(t,t) \neq EV^1(u,u)$ if $t \neq u$ [Enelow and Hinich, 1989]. In our model, if the candidates converge, the expected vote depends on the location they choose. In our square root example, if $t^1 = t^2 = t$, then

$$EV^1 = F(k^1 - k^2),$$

where $k^1 - k^2 = [t(B^{A2} - B^{A1})(b^1)^2/2] - [t(B^{B2} - B^{B1})(b^2)^2]/2$. So, EV^1 is a function of t, unless $B^{A2} = B^{A1}$ and $B^{B2} = B^{B1}$, the sufficient conditions in Theorem 3 for candidate convergence and no contributions.

CANDIDATE EQUILIBRIUM AND FIRM-SPECIFIC POLICY EFFECTS

In our model, we assume two diametrically opposed firms, each of which attempts to help its favorite candidate win the election. Intuition suggests that if contributions have an indirect effect on candidate policy positions, this effect will be to pull the candidates in opposite directions. Austen-Smith [1987] finds this not to be the case; if contributions affect candidate policy positions, the candidates will be pulled in the same direction. We show in this section that Austen-Smith's conclusion may not hold if both candidate-specific and firm-specific policy effects exist.

Austen-Smith [1987] contrasts candidate equilibrium strategies in the absence of firms (denoted t^{1+}, t^{2+}) with equilibrium strategies when firms are introduced into the model (t^{1*}, t^{2*}). He finds that if firms have an effect on candidate strategies, either $t^{k*} < t^{k+}$ or $t^{k*} > t^{k+}$ for both $k = 1, 2$ (Proposition 3). The following theorem establishes that candidates can be pulled in opposite directions by firm contributions only if both candidate-specific and firm-specific policy effects exist. The proof appears in the Appendix. In keeping with Austen-Smith's assumption of a linear probabilistic voting function, we assume a linear distribution function for each voting bloc. The following theorem can be proved for nonlinear distribution functions if candidates converge in the absence of contributions, as they do in our model.

THEOREM 4. *If $B^{A1} = B^{B1}$, $B^{A2} = B^{B2}$, and sgn B^{A1} =sgn B^{A2}, then $t^{k*} < t^{k+}$ or $t^{k*} > t^{k+}$ for both $k = 1, 2$.*

Firm contributions move the candidates in opposite directions only if candidate-specific and firm-specific policy effects exist. In our one group, square root example, with a single ideal point at .5, the candidates will obviously converge to $t^{k+} = .5$ in the absence of firms. Consider, however, the following parameter values: $b^1 = b^2 = 1$, $B^{A1} = B^{B2} = 2$, and $B^{A2} = B^{B1} = 1$. Solving (15) and (16), we find that $t^{1*} = 1/4$ and $t^{2*} = 3/4$. The introduction of firms moves the candidates in opposite directions, with the magnified policy effect of candidate 1 on firm A causing 1 to move closer to A's ideal point, and the magnified policy effect of candidate 2 on firm B having the opposite effect.

An implication from Austen-Smith's [1987] results is that candidate-specific policy effects on voters alone will not cause candidate shifts in opposite directions. Candidate-specific effects on firms must exist for polarization to occur. This is not surprising since voters are not allowed to contribute to the candidates in either his model or ours.

CONCLUSION

We have shown that differential policy effects on firms play an important role in explaining firm contributions to candidates and candidate policy positions. Without candidate-specific policy effects on firms, candidates in a two-candidate election contest will adopt the same policy position and receive zero contributions from the firms. In addition, firm-specific policy effects are necessary for the introduction of contributions to have a polarizing effect on candidate policy positions. Our

comparative statics analysis establishes the relationship between changes in candidate positions and contributions, policy sensitivity and candidate positions, and policy sensitivity and each candidate's expected vote.

Our results suggest several extensions. One obvious possibility is to assume that the k functions depend on contributions made to both candidates, allowing for negative advertising to affect an opponent's expected vote. We might also assume that candidates are allowed to spend contributions on various campaign activities. The candidate must then allocate his resources based on a comparison of the marginal productivity and prices of these different activities (e.g., television ads, direct mail, campaign appearances) given his budget constraint. He might also wish to save some contributions for his next campaign in an effort to deter potential challengers. Theoretical results in this area would be very helpful as a guide for empirical work.

We have assumed that the firm attempts to help its favorite candidate win the election. Candidate anticipation of firm contributions may cause the candidate to adopt a position more to a firm's liking. Still, this indirect connection is not the same as having firms directly buy public policy (as in [Welch, 1974] or private benefits (as in [Denzau and Munger, 1986; or Hinich and Munger, 1988]. In a game where firms and candidates make simultaneous decisions, firms may directly influence candidate decisions.

If firms are buying goods directly, the desire to back a winner can lead them to support both candidates, something which cannot happen in our model. In our model, if candidates are close together, the race may be close but contributions will be very small. In a simultaneous maximization game, contributions may be highest when uncertainty is greatest regarding who will win the election. This is an avenue that may be worth exploring.

Finally, Ingberman's [1989] analysis of subgame perfect equilibrium in a spatial model with contributions and fixed candidate positions suggests a logical extension of our analysis. Cournot-Nash equilibrium may not be subgame perfect. For example, a subgame perfect equilibrium does not allow contributions to a candidate who loses with probability one. Since Cournot-Nash equilibrium depends on marginal, rather than absolute probabilities, this can occur in our model. However, in Ingberman's model, Cournot-Nash equilibrium contributions are subgame perfect if both candidates have positive probability of winning. It will be interesting to see if this conclusion still holds when candidate locations are allowed to vary.

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APPENDIX

PROOF OF THEOREM 1. To show that π^A is strictly concave in c^1, differentiate (3) twice with respect to c^1, recalling that the firms take the candidate positions as given, yielding

$$\pi^A_{c1c1} = r_{c1c1}(B^{A2}t^2 - B^{A1}t^1). \tag{A1}$$

Substituting $\sum N^i F^i(d^i)$ for r, assuming that $B^{A2}t^2 - B^{A1}t^1 > 0$, (A1) is negative if condition (5) holds. In the same way, condition (6) implies that π^B is strictly concave in c^2. In addition π^A is strictly convex in c^2 while π^B is strictly convex in c^1. Continuity of the firm objective functions follows from the continuity of F^i and k^i. Compactness and convexity of the strategy space is assumed, so the conditions of Friedman [1986, Theorem 2.4] are met for existence of at least one equilibrium point.

Uniqueness of the equilibrium follows if the Jacobian of the game is negative quasi-definite [Friedman, 1986, pp. 44–46]. This means that $J + J^T$ is negative definite, where

$$J = \begin{bmatrix} \pi^A_{c1c1} & \pi^A_{c1c2} \\ \pi^B_{c1c2} & \pi^B_{c2c2} \end{bmatrix}.$$

From the strict concavity of π^A in c^1 and π^B in c^2, the terms on the main diagonal of $J+J^T$ are both negative. In addition, $\pi^A_{c1c2} = r_{c1c2}(B^{A2}t^2 - B^{A1}t^1)$ and $\pi^B_{c1c2} = -r_{c1c2}(B^{B2}t^2 - B^{B1}t^1)$. Assuming that $B^{A2}t^2 - B^{A1}t^1 > 0$ and $B^{B2}t^2 - B^{B1}t^1 > 0$, π^A_{c1c2} and π^B_{c1c2} have opposite signs. The determinant of $J + J^T$ is

$$|J + J^T| = -4r_{c1c1}\, r_{c2c2}(B^{A2}t^2 - B^{A1}t^1)(B^{B2}t^2 - B^{B1}t^1) - (r_{c1c2})^2[(B^{A2}t^2 - B^{A1}t^1) - (B^{B2}t^2 - B^{B1}t^1)]^2.$$

If F^i is linear, $r_{c1c2} = 0$, and the second term of $|J + J^T|$ is zero. The second term is also zero if

$$(B^{A2}t^2 - B^{A1}t^1) = (B^{B2}t^2 - B^{B1}t^1).$$

The first term is positive, so in either case, $|J + J^T| > 0$ and $J + J^T$ is negative definite. ∎

PROOF OF THEOREM 2. Given equilibrium in the firm game, we prove condition (7) implies that EV^1 is strictly concave in t^1. Twice differentiating (2) with respect to t^1 yields

$$EV^1_{t1t1} = \sum N^i[f^i_{d_i}(d^i)\,(d^i_{t1})^2 + f^i(d^i)\,d^i_{t1t1}]. \tag{A2}$$

Condition (7) implies that (A2) is negative, so EV^1 is strictly concave in t^1. In the same way, condition (8) implies that EV^2 is strictly concave in t^2. Since each candidate anticipates equilibrium firm contributions, $d^i_{t1} = k^{i1}_{c1}c^1_{t1} - k^{i2}_{c2}c^2_{t1} + u^i_{t1}$, where the partial derivatives of c^1 and c^2 with respect to t^1 incorporate the firms' best response functions.

Each candidate objective function is continuous since it is the sum of continuous functions. Compactness and convexity of the strategy space are assumed so the conditions of Friedman [1986, Theorem 2.4] are met for existence of at least one equilibrium point.

Uniqueness of the equilibrium follows if the Jacobian of the game is negative quasi-definite [Friedman, 1986, pp. 44–46]. This means that $J + J^T$ is negative definite, where

$$J = \begin{bmatrix} EV^1_{t1t1} & EV^1_{t1t2} \\ EV^2_{t1t2} & EV^2_{t2t2} \end{bmatrix}.$$

From the strict concavity of EV^1 in t^1 and EV^2 in t^2, the terms on the main diagonal of $J+J^T$ are both negative. In addition, $EV^1_{t1t2} = -EV^2_{t1t2}$, so $|J + J^T| = 4EV^1_{t1t1}EV^2_{t2t2} > 0$. Thus, $J + J^T$ is negative definite. ∎

PROOF OF THEOREM 3. If $B^{A1} = B^{A2}$ and $B^{B1} = B^{B2}$, then $t^1 = t^2 = t$ implies that

$$EV^1 = \sum N^i F^i(k^{i1}(0) - k^{i2}(0)),$$

regardless of the value of t. Now, assume that $t^{1*} \neq t^{2*}$. Given the strict concavity of EV^1 in t^1,

$$EV^1(t^{1*}, t^{2*}) > EV^1(t^{2*}, t^{2*}).$$

Given the strict convexity of EV^1 in t^2,

$$EV^1(t^{1*}, t^{1*}) > EV^1(t^{1*}, t^{2*}).$$

But, if $B^{A1} = B^{A2}$ and $B^{B1} = B^{B2}$,

$$EV^1(t^{1*}, t^{1*}) = EV^1(t^{2*}, t^{2*}),$$

so $EV^1(t^{1*}, t^{2*}) > EV^1(t^{1*}, t^{2*})$ which is a contradiction. Thus $t^{1*} = t^{2*}$. ∎

PROOF OF THEOREM 4. Define $\nu^1 \equiv \pi^A_{c1}$ and $\nu^2 \equiv \pi^B_{c2}$. In addition, let ν^i_j be shorthand for ν^i_{cj}. Totally differentiating ν^1 with respect to t^1 and t^2 yields the two systems of equations

$$\begin{bmatrix} \nu^1_1 & \nu^1_2 \\ \nu^2_1 & \nu^2_2 \end{bmatrix} \begin{bmatrix} c^1_{t1} \\ c^2_{t1} \end{bmatrix} = \begin{bmatrix} -\nu^1_{t1} \\ -\nu^2_{t1} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \nu^1_1 & \nu^1_2 \\ \nu^2_1 & \nu^2_2 \end{bmatrix} \begin{bmatrix} c^1_{t2} \\ c^2_{t2} \end{bmatrix} = \begin{bmatrix} -\nu^1_{t2} \\ -\nu^2_{t2} \end{bmatrix}.$$

Using Cramer's rule,

$$c_{t1}^1 = \frac{1}{\Delta}(-\nu_{t1}^1 \nu_2^2 + \nu_{t1}^2 \nu_2^1), \tag{A3}$$

$$c_{t2}^1 = \frac{1}{\Delta}(-\nu_{t2}^1 \nu_2^2 + \nu_{t2}^2 \nu_2^1), \tag{A4}$$

$$c_{t1}^2 = \frac{1}{\Delta}(-\nu_1^1 \nu_{t1}^2 + \nu_1^2 \nu_{t1}^1), \quad \text{and} \tag{A5}$$

$$c_{t2}^2 = \frac{1}{\Delta}(-\nu_1^1 \nu_{t2}^2 + \nu_1^2 \nu_{t2}^1), \tag{A6}$$

where $\Delta = \nu_1^1 \nu_2^2 - \nu_2^1 \nu_1^2$.

Candidate 1 and 2's first-order conditions are given by equations (9) and (10), repeated below as (A7) and (A8):

$$EV_{t1}^1 = \sum_{i=1}^k N^i f^i(d^i)[k_{c1}^{i1} c_{t1}^1 - k_{c2}^{i2} c_{t1}^2 + u_{t1}^i] = 0, \tag{A7}$$

$$EV_{t2}^2 = \sum_{i=1}^k N^i f^i(d^i)[k_{c2}^{i2} c_{t2}^2 - k_{c1}^{i1} c_{t2}^1 + u_{t2}^i] = 0. \tag{A8}$$

Since (A7) and (A8) reduce to $\sum N^i f^i(d^i) u_{tk}^i = 0$, $k = 1, 2$, when firms are absent from the model, the effect of firm contributions on candidate locations can be determined by signing

$$Z^{i1} = [k_{c1}^{i1} c_{t1}^1 - k_{c2}^{i2} c_{t1}^2], \quad \text{and} \quad Z^{i2} = [k_{c2}^{i2} c_{t2}^2 - k_{c1}^{i1} c_{t2}^1]$$

at t^{k+}. Setting $B^{A1} = B^{B1} = B^1$, $B^{A2} = B^{B2} = B^2$, and substituting for ν_{tk}^1 and ν_{tk}^2 in (A3) through (A6), we obtain:

$$c_{t1}^1 = \frac{B^1}{\Delta}(r_{c1}\nu_2^2 + r_{c2}\nu_2^1), \qquad c_{t2}^1 = \frac{-B^2}{\Delta}(r_{c1}\nu_2^2 + r_{c2}\nu_2^1),$$

$$c_{t1}^2 = \frac{-B^1}{\Delta}(r_{c2}\nu_1^1 + r_{c1}\nu_1^2), \quad \text{and} \quad c_{t2}^2 = \frac{B^2}{\Delta}(r_{c2}\nu_1^1 + r_{c1}\nu_1^2).$$

Thus,

$$Z^{i1} = \frac{B^1}{\Delta}[k_{c1}^{i1}(r_{c1}\nu_2^2 + r_{c2}\nu_2^1) + k_{c2}^{i2}(r_{c2}\nu_1^1 + r_{c1}\nu_1^2)], \quad \text{and} \quad Z^{i2} = \frac{B^2}{\Delta}[k_{c1}^{i1}(r_{c1}\nu_2^2 + r_{c2}\nu_2^1) + k_{c2}^{i2}(r_{c2}\nu_1^1 + r_{c1}\nu_1^2)].$$

So, if sgn $B^1 = $ sgn B^2, then sgn $Z^{i1} = $ sgn Z^{i2}. ∎

PROOF OF PROPOSITION 1. Given linear F^i, we can derive from the candidates first order conditions

$$\nu_{t1}^1 = -B^{A1}\sum N^i f^i(d^i) w^i k_{c1}^{i1} < 0, \qquad \nu_{t1}^2 = -B^{B1}\sum N^i f^i(d^i) w^i k_{c2}^{i2} < 0,$$

$$\nu_{t2}^1 = B^{A2}\sum N^i f^i(d^i) w^i k_{c1}^{i1} > 0, \qquad \nu_{t2}^2 = B^{B2}\sum N^i f^i(d^i) w^i k_{c2}^{i2} > 0.$$

From the second order conditions, ν_1^1 and $\nu_2^2 < 0$. If F^i is linear, ν_2^1 and $\nu_1^2 = 0$. So equations (A3)–(A6) reduce to

$$\frac{\partial c^1}{\partial t^1} = \frac{-1}{\Delta}[\nu_{t1}^1 \nu_2^2] < 0, \qquad \frac{\partial c^2}{\partial t^1} = \frac{-1}{\Delta}[\nu_1^1 \nu_{t1}^2] < 0,$$

$$\frac{\partial c^1}{\partial t^2} = \frac{-1}{\Delta}[\nu_{t2}^1 \nu_2^2] > 0, \qquad \frac{\partial c^2}{\partial t^2} = \frac{-1}{\Delta}[\nu_1^1 \nu_{t2}^2] > 0. \quad ∎$$

PROOF OF PROPOSITION 2. Let x vary continuously across elections, with $v^i = v^i(x)$ and $v_x^i > 0$. Differentiating the firms first order conditions with respect to x,

$$\nu_x^1 = \Delta \hat{t} \sum N^i w^i f_{di}^i k_{c1}^{i1} \Delta u^i v_x^i, \qquad \nu_x^2 = \Delta \hat{t} \sum N^i w^i f_{di}^i k_{c2}^{i2} \Delta u^i v_x^i.$$

Assume F^i is linear so $f_{di}^i = 0$ and $\nu_x^1 = \nu_x^2 = 0$. At equilibrium, totally differentiate the firms' first order conditions with respect to x. Using Cramer's rule as in the proof of Theorem 4 yields the following equations:

$$\frac{\partial c^1}{\partial x} = \frac{-1}{\Delta}[\nu_x^1 \nu_2^2] = 0, \qquad \frac{\partial c^2}{\partial x} = \frac{-1}{\Delta}[\nu_1^1 \nu_x^2] = 0. \quad ∎$$

PROOF OF PROPOSITION 3. Define

$$\mu^1(t^1,t^2) \equiv \frac{\partial EV^1}{\partial t^1} = \sum N^i f^i(d^i)\{w^i(k_{c1}^{i1}\,c_{t1}^1\,B^{A1} - k_{c2}^{i2}\,c_{t1}^2\,B^{B1}) + v^i u_{t1}^i(t^1)\} = 0,$$

$$\mu^2(t^1,t^2) \equiv \frac{\partial EV^2}{\partial t^2} = -\sum N^i f^i(d^i)\{w^i(k_{c1}^{i1}\,c_{t2}^1\,B^{A2} - k_{c2}^{i2}\,c_{t2}^2\,B^{B2}) - v^i u_{t2}^i(t^2)\} = 0$$

and

$$\mu_1^1 \equiv \frac{\partial^2 EV^1}{(\partial t^1)^2} = \sum N^i\{f_{di}^i(d_{t1}^i)^2 + f^i d_{t1t1}^i\} < 0, \qquad \mu_2^1 \equiv \frac{\partial^2 EV^1}{\partial t^1 \partial t^2} = \sum N^i\{f_{di}^i d_{t2}^i d_{t1}^i + f^i d_{t1t2}^i\},$$

$$\mu_1^2 \equiv \frac{\partial^2 EV^2}{\partial t^1 \partial t^2} = -\sum N^i\{f_{di}^i d_{t2}^i d_{t1}^i + f^i d_{t1t2}^i\}, \qquad \mu_2^2 \equiv \frac{\partial^2 EV^2}{(\partial t^2)^2} = -\sum N^i\{f_{di}^i(d_{t2}^i)^2 + f^i d_{t2t2}^i\} < 0.$$

Although μ_2^1 and μ_1^2 are unsigned, $\mu_2^1 = -\mu_1^2$.

The term $\mu^1(t^1,t^2)$ implicitly defines t^1 as a function of t^2 and $\mu^2(t^1,t^2)$ implicitly defines t^2 as a function of t^1, i.e., the candidates' respective reaction functions. By total differentiation of μ^1 and μ^2,

$$s^1 \equiv \frac{dt^1}{dt^2} = -\frac{\mu_2^1}{\mu_1^1}, \qquad s^2 \equiv \frac{dt^2}{dt^1} = -\frac{\mu_1^2}{\mu_2^2},$$

the slopes of candidates 1 and 2's reaction functions. Note that $\operatorname{sgn} s^1 = -\operatorname{sgn} s^2$.

Let x vary continuously across races, with $v^i = v^i(x)$ and $v_x^i > 0$. Assume F^i linear. c^j ($j = 1,2$) is, therefore, independent of x (Proposition 2) so that

$$\mu_x^1 = \sum N^i f^i v_x^i\, u_{t1}^{i1}, \qquad \mu_x^2 = \sum N^i f^i v_x^i\, u_{t2}^{i2},$$

μ_x^1 and μ_x^2 are weighted averages of u_{t1}^{i1} and u_{t2}^{i2} over all i. When $\mu_x^1 > 0$, and $u^i(t) = -(t - X^i)^2$, t^1 is left of the average group ideal point $\sum N^i f^i v_x^i X^i / \sum N^i f^i v_x^i$ (where X^i is group i's ideal point). When $\mu_x^1 < 0$, t^1 is to the right of this point, and when $\mu_x^1 = 0$, t^1 equals this average ideal point. Similarly for μ_x^2 and t^2.

Suppose $t^1 < t^2$ and u^i is strictly concave. Then only 5 cases or types are possible for each group i:

(1) $u_{t1}^{i1} > 0$, $u_{t2}^{i2} > 0$ and $u_{t1}^{i1} > u_{t2}^{i2}$;

(2) $u_{t1}^{i1} > 0$, $u_{t2}^{i2} = 0$;

(3) $u_{t1}^{i1} > 0$, $u_{t2}^{i2} < 0$:

(4) $u_{t1}^{i1} = 0$, $u_{t2}^{i2} < 0$;

(5) $u_{t1}^{i1} < 0$, $u_{t2}^{i2} < 0$ and $u_{t1}^{i1} > u_{t2}^{i2}$.

Let $U^{j1} = \sum_{i \in j} N^i f^i v_x^i u_{t1}^{i1}$ ($j = 1,\ldots,5$) be the terms of μ_x^1 associated with the groups belonging to type j, so that $\mu_x^1 = \sum z^j U^{j1}$, where z^j is either 0 or 1, depending on whether a group of type j exists. Defining U^{j2} similarly, $\mu_x^2 = \sum z^j U^{j2}$.

Given $t^1 < t^2$ and u^i strictly concave, the following are possible sign combinations for μ_x^1 and μ_x^2 (respectively):

(a) $(+,+)$;   (d) $(0,-)$;

(b) $(+,0)$;   (e) $(-,-)$.

(c) $(+,-)$;

Existence follows by assuming all groups are of one type.

Given $t^1 < t^2$ and u^i strictly concave, the following are not possible sign combinations for μ_x^1 and μ_x^2 (respectively):

(f) $(-,+)$;   (h) $(0,+)$;

(g) $(-,0)$;   (i) $(0,0)$.

From the definitions of types 1 and 5,

$$z^1 U^{11} + z^5 U^{51} > z^1 U^{12} + z^5 U^{52}.$$

From the definitions of the remaining types,

$$-(z^2 U^{21} + z^3 U^{31} + z^4 U^{41}) < 0 \qquad \text{and} \qquad -(z^2 U^{22} + z^3 U^{32} + z^4 U^{42}) > 0.$$

Assume the existence of case (f). Then

$$z^1 U^{11} + z^2 U^{51} < -(z^2 U^{21} + z^3 U^{31} + z^4 U^{41}) \quad \text{and} \quad z^1 U^{12} + z^2 U^{52} > -(z^2 U^{22} + z^3 U^{32} + z^4 U^{42}).$$

Combining the five inequalities,

$$-(z^2 U^{21} + z^3 U^{31} + z^4 U^{41}) > z^1 U^{11} + z^2 U^{51} > z^1 U^{12} + z^2 U^{52} > -(z^2 U^{22} + z^3 U^{32} + z^4 U^{42}).$$

Hence,

$$-(z^2 U^{21} + z^3 U^{31} + z^4 U^{41}) > -(z^2 U^{22} + z^3 U^{32} + z^4 U^{42}).$$

But this is impossible because the left hand side of the inequality is negative and the right hand side is positive. So case (f) cannot exist. Proofs of cases (g)–(i) are virtually identical and are omitted for brevity.

In equilibrium, totally differentiate the candidates' first order conditions with respect to x. Using Cramer's rule and the definitions of the reaction functions, we obtain

$$\frac{\partial t^1}{\partial x} = \frac{-1}{\Delta}[\mu_x^1 \mu_2^2 + \mu_1^1 s^1 \mu_x^2], \qquad \frac{\partial t^2}{\partial x} = \frac{-1}{\Delta}[\mu_x^1 s^2 \mu_2^2 + \mu_1^1 \mu_x^2].$$

CASE (a). If $s^1 > 0$ and $s^2 < 0$, $\frac{\partial t^1}{\partial x} > 0$ and $\frac{\partial t^2}{\partial x}$ is ambiguous; if $s^1 < 0$ and $s^2 > 0$ then $\frac{\partial t^1}{\partial x}$ is ambiguous and $\frac{\partial t^2}{\partial x} > 0$. If $s^1 > 0$, and $u^i = -(t - X^i)^2$, then

$$t^1 < \frac{\sum N^i f^i v_x^i X^i}{\sum N^i f^i v_x^i}.$$

If $s^2 > 0$,

$$t^2 < \frac{\sum N^i f^i v_x^i X^i}{\sum N^i f^i v_x^i},$$

so in either case at least one candidate moves toward $\sum N^i f^i v_x^i X^i / \sum N^i f^i v_x^i$. Examination of cases (b)–(e) proceeds similarly with identical conclusions.   ∎

PROOF OF PROPOSITION 4. Let x vary continuously across elections, with $v^i = v^i(x)$ and $v_x^i > 0$. Then (using the envelope theorem)

$$\frac{\partial EV^1}{\partial x} = \sum N^i f^i [w^i (k_{c1}^{i1} c_x^1 - k_{c2}^{i2} c_x^2) + \Delta u^i v_x^i], \qquad \text{and} \qquad \frac{\partial EV^2}{\partial x} = -\frac{\partial EV^1}{\partial x}.$$

With linear F^i, $c_x^j = 0$ ($j = 1, 2$) (Proposition 2) so

$$\frac{\partial EV^1}{\partial x} = \sum N^i f^i \Delta u^i v_x^i,$$

The points in the proposition follow immediately.   ∎