Technical Appendix for “Courts to the Rescue?”

Ages and Actuarial Death Probabilities

Here are the ages of the Circuit’s seven Democrats:

Judge (Democrats)	Age	Actuarial Death Probability
David Tatel	76	0.04
Judith Rogers	78	0.03
Merrick Garland	65	0.02
Robert Wilkins	54	0.01
Cornelia Pillard	57	0.01
Sri Srinivasan	51	0.01
Patricia Millett	54	0.004

Table 1. The Grim Reaper Views the D.C. Circuit

Here is the age distribution and associated annual death probabilities of the Democrats on the Supreme Court

Justice (Democrats)	Age	Actuarial Death Probability
Ruth Bader Ginsburg	85	0.07
Stephen Breyer	79	0.05
Sonia Sotomayor	63	0.01
Elena Kagan	57	0.01

Table 2. Age and Actuarial Death Probabilities of Democratic Supreme Court Justices

The probabilities come from on-line actuarial tables from Social Security Administration.

Panel Composition Probabilities

The are 11 judges, three are drawn randomly to form a panel. Given *N* Democrats on the bench, what is the probability a panel will be Democrat-dominated (at least two Democrats)?

To solve this problem I make extensive use of the binomial theorem, which indicates the number of ways of choosing *r* objects from *n* objects. The formula is $\frac{n!}{r!(n-r)!} = \binom{n}{r}$ where “n!” (read, “n factorial”) means $n \times (n-1) \times (n-2) \ldots (n-(n-1))$.

First note that there are $\binom{11}{3} = 165$ possible 3-member panels from a bench of 11. That is, number the judges from 1 to 11. One panel is {1,2,3}, another is {1,2,4} and so on. There are 165 of these possible panels and each one has the same probability of being drawn, namely, 1/165.

Now consider panels that are composed DDD. Given *N* Democratic judges on the bench, how many possible panels are DDD? This is just $\binom{N}{3}$ panels. For instance, if N = 0, 1, or 2, there can't be any DDD panels. If N = 3, there is exactly one DDD panel, so the probability of drawing it is $\frac{1}{165} = .006$. But if there are four Democratic judges, we have $\binom{4}{3} = 4$ possible DDD panels so the probability of getting one is $\frac{4}{165} = .02$. To be clear, the formula for calculating the probability of DDD panels is $\frac{\binom{N}{3}}{\binom{11}{3}} = \frac{\binom{N}{3}}{165}$.

Now consider panels with two Democrats and one Republican. How many possible panels are like this? This is slightly trickier but not much. First, given *N* Democrats, how many ways are there to draw 2 person DD panels? This is just $\binom{N}{2}$. Then, for each one of these panels, there are $11 - N$ ways to fill the Republican slot. So there are $\binom{N}{2}(11 - N)$ 2-D 1-R panels. For example, if there are 3 Democrats and 8 Republicans on the bench, there are $\binom{3}{2} = 3$ ways to arrange the Democrats into groups of two. Then, for each of these groups, there are 8 ways to fill out the panel with a Republican. So there are $\binom{3}{2}(11 - 3) = 3 \times 8 = 24$ possible DDR panels, and the probability of getting one of these is just $\frac{24}{165} = .145$. To be clear, the formula for calculating the probability of DDR panels is $\frac{\binom{N}{2}(11-N)}{\binom{11}{3}} = \frac{\binom{N}{2}(11-n)}{165}$.

To get the probability of a D-Dominated panel, we add the probability of DDD panels and the probability of DDR panels to get

$$\frac{\binom{N}{3} + \binom{N}{2}(11 - N)}{\binom{11}{3}} = \frac{N(33N - 2N^2 - 31)}{990} \qquad (1)$$

In the text, I was particularly concerned about this probability when N = 7, 6, and 5. Using Equation (1) these probabilities as .72, .58, and .28.

The following figure indicates the probability of a Democrat-dominated panel for any number of Democrats on the 11 member bench.

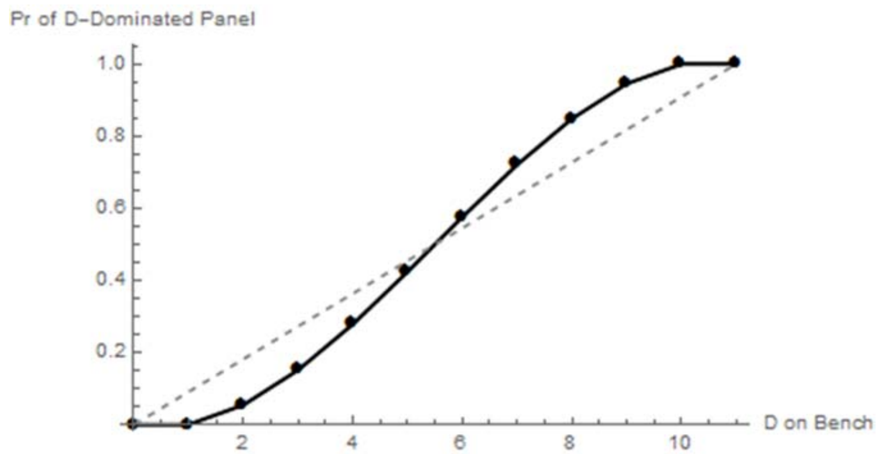

Figure 1. The Probability of a Democrat-Dominated Panel Given the Number of Democrats on the D.C. Circuit. The solid black line shows the probabilities calculated using Equation (1). The dashed gray line shows a probability strictly proportional to fraction of the bench, e.g., N=6 yields 6/11. The use of panels tends to magnify slightly the advantage of being in the majority, and the penalty from being in the minority.

D.C. Circuit Transition Probabilities

The annual death probabilities are actually $0.0398820, .0345820, .0158260, .0071590, .0055280, .0054730, .004423$. I use these in calculations.

The probability that none of the seven die in a single year is $\prod_{i=1}^{7}(1 - x_i) = .89$. Obviously, the chance that at least one dies is the reciprocal probability, 11%.

The chance that all live for two years is $\prod_{i=1}^{7}(1 - x_i)^2 = .80$. The probability that at least one dies is the reciprocal probability, .20.

The probability that two or more die in one year is: 1 minus (the probability that none dies plus the probability that exactly one dies). We have already calculated the probability that none dies (.89). The probability that exactly one dies is $\sum_{i=1}^{7}\prod_{j\neq i}(1 - x_j)$. Using the above probabilities I calculate this to be .09. So the one year probability that two or more dies is $1 - .89 - .09 = .01$.

The chance that two or more die over two years is a bit more complex. It is the chance that two or more die in the first year, plus the chance that exactly one dies in the first year and one or more dies in the second year, plus the chance that none dies in the first year and two or more dies in the second year. We have already calculated these probabilities, and combining them yields .037.

Supreme Court Transition Probabilities

The annual death probabilities are actually $0.0738280, .0531230, .0083390, .005528$. I use these in calculations.

The probability that none of the four die in a single year is $\prod_{i=1}^{4}(1-x_i) = .865$. Obviously, the chance that at least one dies is the reciprocal probability, 13.5%.

The probability that none of the four dies in two years is $\prod_{i=1}^{4}(1-x_i)^2 = .75$. The probability that at least one dies over the 2-year period is the reciprocal, .25.

Let's extend the analysis to six years. Then we have $\prod_{i=1}^{4}(1-x_i)^6 = .42$. And the probability that at least one dies is 58%.

However, this quick estimate does not take into account that death probabilities rise each year for each survivor. The formula then becomes $\prod_{i=1}^{4}\prod_{j=1}^{6}\left(1-x_{ij}\right)$. I omit the calculation with the 24 different annual probabilities, but the net result is, the probability that all live is 31%. And, the probability that at least one dies is 69%.

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